



## CONSTITUTIVE DESCRIPTION OF WORK- AND SHOCK-HARDENED COPPER

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(Received September 29, 1993)  
 (Revised December 15, 1993)

### Introduction

Constitutive models are an essential part of large-scale computational codes which describe material behavior, since they provide the relationship between stress ( $\sigma$ ), strain ( $\epsilon$ ), strain rate ( $\dot{\epsilon}$ ), and temperature ( $T$ ). However, plastic deformation is an irreversible and path-dependent process, and a number of parameters affect the development of the deformation structure and, as a consequence, the mechanical response. The stress state, strain rate, and temperature affect the evolution of the microstructure, and the strain, current temperature and strain rate alone are frequently not sufficient to describe it. The constitutive equations that have been developed fall broadly into two groups:

- a) Empirical constitutive equations. These equations have work hardening, thermal softening, and strain-rate hardening terms. Work hardening is usually described by a Ludwik [1] power hardening term,  $\epsilon^n$ , where  $n$  is a work hardening coefficient. The strain rate sensitivity is represented by a number of functions:  $(\dot{\epsilon}/\dot{\epsilon}_0)^M$ ;  $(1 + a\dot{\epsilon})^M$ ;  $(1 + M \ln \dot{\epsilon}/\dot{\epsilon}_0)$ , where  $\dot{\epsilon}_0$ ,  $a$  and  $M$  are constants. The effect of temperature on the flow stress is represented by exponential or power-law functions. Examples of these constitutive models are the equations used by Littonski [2], Vinh *et al.* [3], Klopp *et al.* [4], and Johnson and Cook [5].
- b) Microstructurally-based constitutive equations. These equations are based on the micromechanics of plastic deformation and are rooted in the thermally-activated motion of dislocations [6]. The separation of short- and long-range barriers as well as their shape yields the strain, strain-rate, and temperature dependence of the flow stress. The equations developed by Kocks *et al.* [7], Hoge and Mukherjee [8], Follansbee and Kocks [9], Zerilli and Armstrong [10], and Klepaczko [11] use this approach. The thermal softening term in these equations is a continuous function.

Work-hardened metals may undergo dynamic recrystallization if the deformation temperature is in the range of 0.4 to 0.5  $T_m$ , where  $T_m$  is the absolute melting temperature. This dynamic recrystallization is accompanied by marked changes in the thermomechanical response of the material, which are not addressed by the constitutive equations discussed above. In high-strain-rate adiabatic deformation, the thermal energy due to plastic work often brings the metal to temperatures  $>0.4 T_m$ , and Meyers and Pak [12] and Chokshi and Meyers [13] have proposed that dynamic recrystallization plays an important role in shear-band formation and the extended tensile ductility undergone by copper shaped charges, respectively. More recent research by Meyers *et al.* [14], Murr *et al.* [15], and Gurevitch *et al.* [16] demonstrated clearly the importance of recrystallization in high-strain-rate deformation. Systematic reports of studies on copper and titanium will appear elsewhere [17, 18].

The objective of this note is to develop a procedure for incorporating these important microstructural evolution elements into the constitutive equations that describe plastic deformation.

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### Experimental Procedures

OFHC copper (99.99% pure) was used in this study. It was received in a cold worked condition (25 pct reduction) and was subsequently shock hardened to a pressure of 50 GPa, corresponding to a total true effective strain of 0.276. Cylindrical specimens of the work-hardened and shock-hardened conditions were tested in compression at a strain rate of  $10^{-3} \text{ s}^{-1}$  and different temperatures. The specimens were placed in the furnace after the test temperature had already been stabilized: typically, it took five minutes for the specimen to equilibrate to the test temperature. The experimental procedures are described elsewhere [17] in detail.

The effect of temperature on the compressive stress-strain response of Cu in the cold-worked and shocked conditions is shown in Figure 1. There is a gradual decrease in the flow

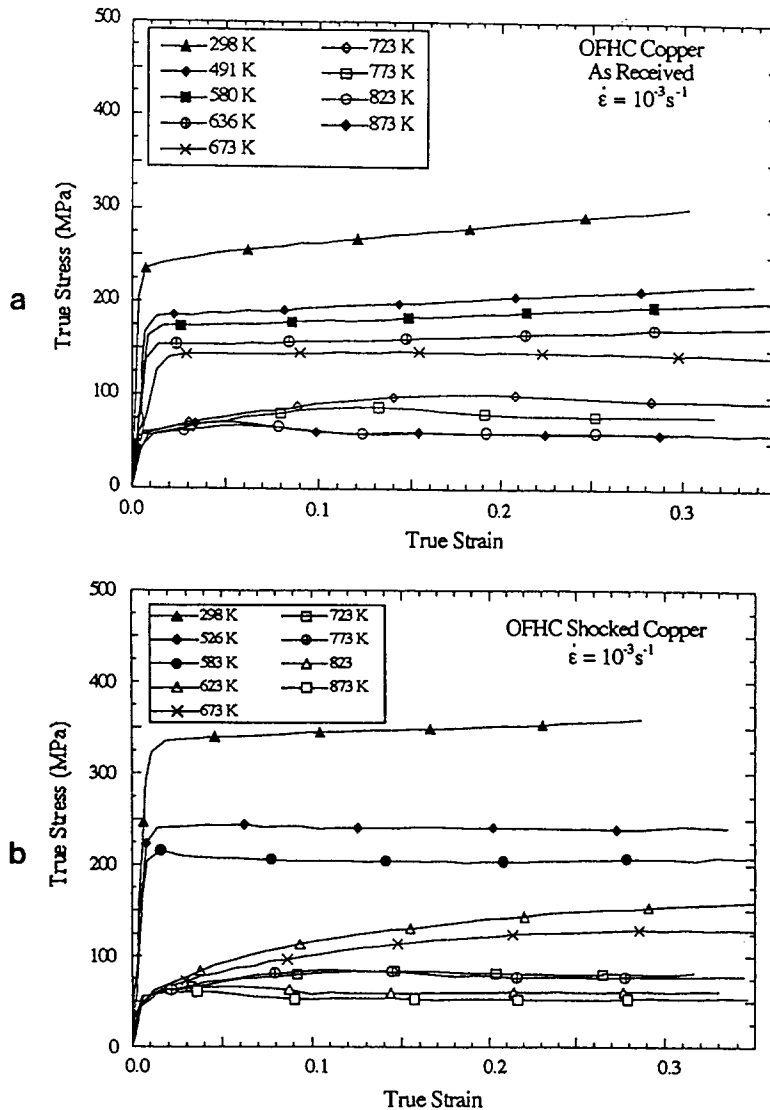


Figure 1. Compressive stress-strain curves for (a) cold-worked and (b) shocked copper as a function of temperature.

stress in these two conditions, as the temperature is increased. At temperatures greater than 583 K (for the shocked material) and greater than 673 K (for the cold worked material) there is a distinct drop in the flow stress. This drop corresponds to the dynamic recrystallization temperatures, as is shown in [17]. The recrystallization temperature is known to decrease with the amount of cold work [19]; the results of Figure 1 corroborate this.

### Constitutive Description

The Johnson-Cook [5] equation is one of the most common and simplest constitutive models and, therefore, it was used in the current study. The original form of the equation is:

$$\sigma = (\sigma_0 + B\epsilon^n) \left(1 + C \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right) \left[1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right] \quad (1)$$

$\sigma_0$ ,  $B$ ,  $n$ ,  $C$ , and  $m$  are parameters determined experimentally;  $T$ ,  $T_r$ , and  $T_m$  are the current, reference (usually, 298 K) and melting (1356 K) temperatures, respectively;  $\dot{\epsilon}_0$  is the reference strain rate ( $10^{-3}\text{s}^{-1}$  in the analysis below). In this equation, the thermal softening is gradual and cannot accommodate a sudden flow stress change due to dynamic recrystallization or phase transformation. Indeed, preliminary attempts at fitting the present experimental data into the Johnson-Cook equation [5] yielded a very poor correlation. The simplest manner by which this change in flow stress, accompanying dynamic recrystallization, can be incorporated is by means of a reducer function  $H(T)$ :

$$\sigma = (\sigma_0 + B\epsilon^n) \left(1 + C \ln \frac{\dot{\epsilon}}{\dot{\epsilon}_0}\right) \left[1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m\right] H(T) \quad (2)$$

This reducer function incorporates a unit step function of temperature, defined as:

$$u(T) = \begin{cases} 0 & \text{when } T < T_C \\ 1 & \text{when } T > T_C \end{cases} \quad (3)$$

where  $T_C$  is the temperature at which the critical phenomenon (dynamic recrystallization, phase transformation, etc.) occurs. Thus,  $H(T)$  has the form:

$$H(T) = \frac{1}{1 - (1 - \bar{H})u(T)} \quad (4)$$

A suitable value can be assigned for  $\bar{H}$ , a reducer constant that expresses the fractional change in flow stress at the temperature of the critical phenomenon,

$$\bar{H} = \frac{(\sigma_f)_{\text{rec}}}{(\sigma_f)_{\text{def}}} \quad (5)$$

where  $(\sigma_f)_{\text{rec}}$  and  $(\sigma_f)_{\text{def}}$  are the flow stresses corresponding to the situation just prior to and after recrystallization, respectively. The reduction in flow stress caused by dynamic recrystallization is a direct result of the mechanism of annihilation of the dislocation structure produced by plastic deformation. Jonas *et al.* [20], Derby and Ashby [21], and Derby [22] address some of the issues. In essence, one has a new steady-state grain size,  $d$ , and a steady-state dislocation density when the rate of dislocation generation and annihilation are equal. Components are strain-rate and temperature dependent. Derby and Ashby [21] found that the recrystallized grain size is related to the strain rate by:

$$d \propto \dot{\epsilon}^{-1/2} \quad (6)$$

Chokshi and Meyers [13] used this relationship to explain the very fine recrystallized grain size obtained after high strain-rate deformation of copper. The flow stress of the recrystallized material can be expressed as:

$$(\sigma_f)_{rec} = \sigma_0 + K_1 \rho_{ss}^{\frac{1}{2}} + K_2 \epsilon^{\frac{1}{4}} \tag{7}$$

The above expression assumes Taylor hardening and a Hall-Petch relationship.  $\sigma_0$  is a temperature dependent friction stress,  $K_1$  and  $K_2$  are parameters,  $\rho_{ss}$  is the steady state dislocation density. It is clear that Eqn. (7) contains a number of parameters that have to be experimentally determined.

Using Equations (2) and (4), and experimentally obtaining the parameters  $\sigma_0$ , B, n, C, and m, it is possible to obtain a satisfactory description of the stress-strain response of copper as a function of temperature and strain rate. Based on the experimental data, the dynamic recrystallization temperatures were taken as 700 and 600 K, for the cold worked and shocked conditions, respectively. The flow stress at a plastic strain equal to 0.3 as a function of temperature is plotted in Figures 2(a) and 2(b) for the work-hardened and shocked conditions, respectively. Proper adjustment of the parameters and selection of the reducer constant H

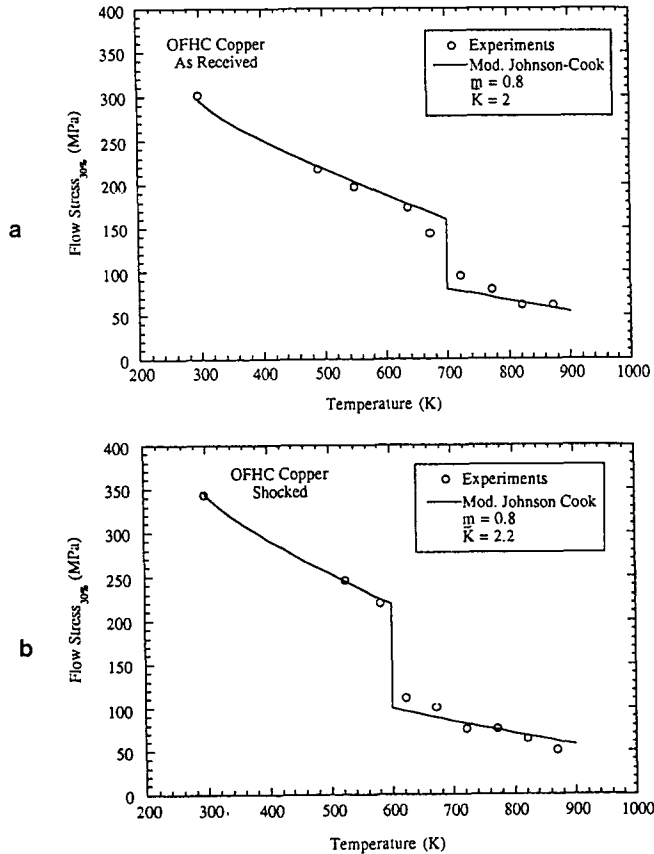


Figure 2. Experimentally determined and computed flow stress (at a plastic strain of 0.3) as a function of temperature for (a) cold-worked and (b) shocked copper.

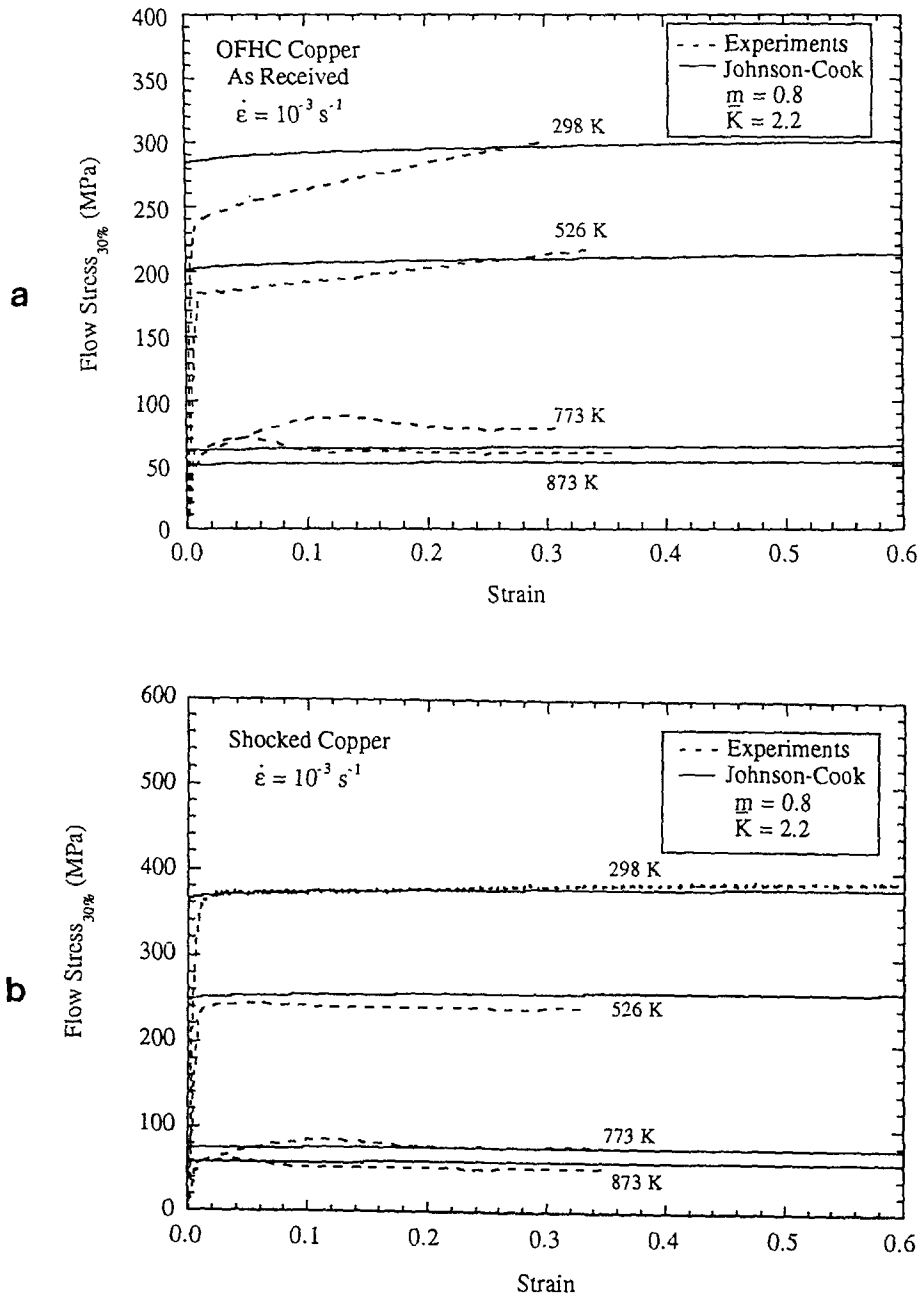


Figure 3. Comparison of computed and experimentally determined stress-strain curves as a function of temperature for (a) cold-worked and (b) shocked copper.

enabled a good fit that correctly represented the sudden flow stress decrease at TC. Application of the modified Johnson-Cook equation to the entire flow stress-strain curves, shown in Figure 3(a) and 3(b) for the work-hardened and shocked conditions, respectively, demonstrates that satisfactory correlation can be obtained between experimentally determined and calculated results.

It is clearly possible to obtain a good fit between experimental results and an empirical model, with a suitable choice of parameters. It is always necessary to exercise caution in extrapolating an empirical equation beyond the range over which experimental data have verified the equation, in view of the microstructural changes occurring that are not incorporated into the formalism. The present study has demonstrated that it is possible to account for the occurrence of dynamic recrystallization by modifying the Johnson-Cook equation using a reducer function. It is suggested that the same form of the modifying function can be applied to other constitutive equations, as well as to other microstructural changes such as phase transformations. If these changes occur over a broad temperature range, then an appropriate function can be incorporated to modify the constitutive equation.

### **Conclusions**

Experimental results on cold-worked and shock-hardened copper reveal the occurrence of dynamic recrystallization at temperatures of ~700 and 600 K, respectively. The available empirical constitutive equations do not generally account for such microstructural changes, and this leads to a poor fit between the experimental results and the predictions. A modified constitutive equation incorporating a reducer function to represent dynamic recrystallization was applied to work hardened and shock-hardened copper, providing a much improved fit.

### **Acknowledgement**

This research was supported by the U. S. Army Research Office University Research Initiative Program (Contract No. DAAL03-86-K-0169), by the National Science Foundation (Grant MSS-902167), and by the Brazilian Government through the National Council for Research.

### **References**

1. P. Ludwik, *Phys. Z.*, 10, 411 (1908).
2. J. Littonski, *Bull. Pol. Acad. Sci.*, 25, 7 (1977).
3. T. Vinh, M. Afzali, and A. Roche, in *Mechanical Behaviour of Materials*, Eds. K. J. Miller and R. F. Smith, Pergamon Press, 633 (1979).
4. R. W. Klopp, R. J. Clifton, and T. G. Shawki, *Mech. Mat.*, 4, 375 (1985).
5. G. R. Johnson and W. H. Cook, in *Proc. 7th Intl. Sym. on Ballistics*, The Hague, Netherlands, 541, (1983).
6. R. Becker, *Z. Phys.*, 26, 919 (1925).
7. U. F. Kocks, A. S. Argon, and M. F. Ashby, *Prog. Mats. Sci.*, 19, 1 (1975).
8. K. G. Hoge and A. K. Mukherjee, *J. Mat. Sci.*, 12, 1666 (1977).
9. P. S. Follansbee and U. F. Kocks, *Acta Met.*, 36, 81 (1988).
10. F. J. Zerilli and R. W. Armstrong, *J. Appl. Phys.*, 61, 1816 (1987).
11. J. R. Klepaczko, *J. de Physique*, 9, 49 (1988).
12. M. A. Meyers and H.-r. Pak, *Acta Met.*, 34, 2493 (1986).
13. A. H. Chokshi and M. A. Meyers, *Scripta Met. Mat.*, 24, 605 (1990).
14. M. A. Meyers, L. W. Meyer, K. S. Vecchio, and U. R. Andrade, *J. de Physique III*, Vol. 1, C3-11 (1991).
15. L. E. Murr, H. K. Shih, C.-S. Niou, and L. Zernow, *Scripta Met. Mat.*, 29, 567 (1993).
16. A. C. Gurevitch, L. E. Murr, H.-K. Shih, C.-S. Niou, A. H. Advani, D. Manuel, and L. Zernow, *Materials Char.*, 30, 201 (1993).
17. U. R. Andrade, M. A. Meyers, K. S. Vecchio, and A. H. Chokshi, *Acta Met. Mat.*, submitted (1993).
18. M. A. Meyers, G. Subhash, B. Kad, and L. Prasad, *Mech. Mats.*, submitted (1993).
19. R. E. Reed-Hill, *Physical Metallurgy Principles*, 2nd Ed., Van Nostrand, p. 288 (1973).
20. B. Derby and M. F. Ashby, *Scripta Met.*, 21, 879 (1987).
21. B. Derby, *Acta Met.*, 39, 955 (1991).
22. J. J. Jonas, C. M. Sellars, and W.J. McG. Tegart, *Metall. Rev.*, 14, 130 (1969).